# ST.ANNE'S

**COLLEGE OF ENGINEERING AND TECHNOLOGY**

(An ISO 9001:2015 Certified Institution)

Anguchettypalayam, Panruti – 607106.

**QUESTION BANK**

**PERIOD:** AUG -DEC-2022 **BATCH**: 2021 – 2025

**BRANCH:** EEE **YEAR/SEM:** II/03

**SUB CODE/NAME:** MA3303 –PROBABILITY AND COMPLEX FUNCTION

## UNIT I – PROBABILITY AND RANDOM VARIABLES

### PART – A

1. Define random variable.
2. X and Y are independent random variables with variances 2 and 3. Find the variance of 3X +4Y.
3. Let X be a R.V with E[X]=1 and E[X(X-1)]=4 . Find var X and Var(2-3X).
4. The number hardware failures of a computer system in a week of operations as the following pmf:

| Number of failures: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Probability : | 0.18 | 0.28 | 0.25 | 0.18 | 0.06 | 0.04 | 0.01 |

   Find the mean of the number of failures in a week

5. A continuous random variable X has the probability density function given by $f(x)=3x^2, 0 \le x \le 1$. Find K such that P(X > K)= 0.5
6. A random variable X has the pdf f(x) given by $f(x)=\begin{cases} Cxe^{-x}, x>0 \\ 0, x \le 0 \end{cases}$. Find the value of C and c.d.f of X.
7. The cumulative distribution function of a random variable X is F(x)= $[1-(1+x)e^{-x}], x>0$. Find the probability density function of X.
8. Is the function defined as follows a density function? $f(x)=\begin{cases} 0, x<2 \\ \frac{1}{18}(3+2x), 2 \le x \le 4 \\ 0, x>4 \end{cases}$
9. Let X be a R.V with p.d.f given by $f(x)=\begin{cases} 2x, 0<x<1 \\ 0, otherwise \end{cases}$. Find the pdf of Y =(3X +1).
10. Find the cdf of a RV is given by $F(x)=\begin{cases} 0, x<0 \\ \frac{x^2}{16}, 0 \le x \le 4 \\ 1, 4<x \end{cases}$ and find P(X>1/X<3).
11. A continuous random variable X that can assume any value between x = 2 and x = 5 has a density function given by f(x) = K(1 + x). Find P[X<4].
12. The first four moments of a distribution about x = 4 are 1, 4, 10 and 45 respectively. Show that the mean is 5, variance is 3, $\mu_3=0$ and $\mu_4=26$.
13. Define moment generating function.

**14.** Find the moment generating function for the distribution where $f(x)=\begin{cases}\frac{2}{3}, x=1\\ \frac{1}{3}, x=2\\ 0, otherwise\end{cases}$.

**15.** For a binomial distribution mean is 6 and S.D is $\sqrt{2}$. Find the first two terms of the distribution.

**16.** Find the moment generating function of binomial distribution.

**17.** The mean of a binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution

**18.** If X is a Poisson variate such that P(X=2) = 9P(X=4) +90P(X=6),find the variance.

**19.** Write the MGF of geometric distribution.

**20.** One percent of jobs arriving at a computer system need to wait until weekends for scheduling , owing to core-size limitations. Find the probability that among a sample of 200 jobs there are no job that have to wait until weekends.

**21.** Show that for the uniform distribution $f(x)=\frac{1}{2a}, -a<x<a$ the moment generating function about origin is $\frac{\sinh at}{at}$.

**22.** If X is a Gaussian random variable with mean zero and variance $\sigma^2$, find the probability density function of $Y=|X|$.

**23.** A random variable X has p.d.f $f(x)=\begin{cases}e^{-x}, x>0\\ 0, x<0\end{cases}$. Find the density function of $\frac{1}{x}$

**24.** State Memoryless property of exponential distribution.

**25.** The mean and variance of binomial distribution are **5** and **4**. Determine the distribution.

**26.** For a binomial distribution mean is **6** and S.D is $\sqrt{2}$ . Find the first of the distribution.

**27.** What are the limitations of Poisson distribution.

**28.** A random variable X is uniformly distributed between 3 and 15.Find mean and variance.

**29.** A continuous random variable X has a p.d.f given by $f(x)=\begin{cases}\frac{3}{4}(2x-x^2), 0<x<2\\ 0 \quad , otherwise\end{cases}$ .Find $p(x>1)$

**30.** Let X be the random variable which denotes the number of heads in three tosses of a fair coin. Determine the probability mass function of X.

**31.** If $f(x)=\begin{cases}ke^{-x}, x>0\\ 0, otherise\end{cases}$ is p.d.f of a random variable X ,then the value of K.

**32.** Find the mean and variance of the discrete random variable X with the p.m.f $p(x)=\begin{cases}\frac{1}{3}, x=0\\ \frac{2}{3}, x=2\end{cases}$

**33.** A random variable X has c.d.f $F(x)=\begin{cases}0 \quad , x<1\\ \frac{1}{2}(x-1), 1\le x\le 3\\ 0 \quad , x>3\end{cases}$ . Find the p.d.f of X and the expected value of X.

# PART – B

## FIRST HALF (All are 8- marks)

(A) DISCRETE DISTRIBUTION:-

***I- Binomial distribution***

1. ***The moment generating function of a binomial distribution and find mean and variance (i.e)*** $p(x) = \begin{cases} nc_x p^x q^{n-x}, & x = 0,1,2,3 \ldots\ldots \\ 0 & ,otherwise \end{cases}$
2. The probability of a bomb hitting a target is $\frac{1}{5}$. Two bombs are enough to destroy a bridge. If six bombs are aimed at the bridge, find the probability that the bridge is destroyed?
3. If 10% of the screws produced by an automatic machine are defective find the probability that out of 20 screws selected at random, there are
   (i) exactly 2 defective (ii) at most 3 defective (iii) at least 2 defective

   (iv) between 1 & 3 defective
4. the probability of a man hitting a target is $\frac{1}{4}$ . If he fires 7- times (i) what is the probability of his hitting the target at least twice? (ii) How many times must he fire so that the probability of hitting the target at least once is greater than $\frac{2}{3}$.
5. A machine manufacturing screws is known to probability 5% defective in a random sample of 15 screws, what is the probability that there are (i) exactly 3 defective (ii) not more than 3 defectives.
6. Five fair coins are flipped. If the outcomes are assumed independent find the probability mass function of the number of heads obtained.

**II- Poisson distribution**

1. ***The moment generating function of a Poisson distribution and find mean and variance (i.e)*** $P(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & ,x = 0,1,2,\ldots\ldots\ldots \\ 0 & ,otherwise \end{cases}$
2. ***Derive the Poisson distribution as a limiting case of binomial distribution***.
3. If 3% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective.



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**4.** The no.of monthly breakdown of a computer is a random variable having a Poisson distribution with mean equal to 1.8, Find the probability that this computer will function a month (i) without a breakdown (ii) with only one breakdown (iii) with at least one breakdown .

**5.** If X is a Poisson variate such that $p(x = 1) = \frac{3}{10}$ and $p(x = 2) = \frac{1}{5}$, find $p(x = 0)$ $and$ $p(x = 3)$.

**6.** If X is a Poisson variate such that $P\ (X = 2) = 9\ P(X = 4) + 90\ P(X = 6)$ find the mean and variance.

## III- <u>Geometric distribution</u>

***1. The moment generating function of a Geometric distribution and find mean and variance (i.e) $p(x) = q^{x-1}P\ \ , x = 1, 2, 3, \dots\dots\ \ where\ q = 1 - p$.***

**2.** ***State and prove the memory less property of the geometric distribution***.

**3.** Suppose that a trainee soldier shoots a target in a independent fashion. If the probability that the target is shot on any one shot is 0.8. (i) what is the the probability that the target would be hit on $6^{th}$ attempt (ii) what is probability that it takes him less than 5 shots (iii) what is probability it takes him even no. of shots.

**4.** If the probability that a target is destroyed on any one shot is 0.5, what is the probability that it would be destroyed on $6^{th}$ attempt?

**5.** If the probability that an applicant for a driver license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (i) on the $4^{th}$ trial. (ii) in fewer than 4 trials.

## <u>Problems on Discrete random variables:-</u>

**1.** If the probability mass function of a random variable X is given by $P[X = x] = kx^3, x = 1,2,3,4,$ (i) find the value of K.(ii) $P\left[\left(\frac{1}{2} < X < \frac{5}{2}\right)/X > 1\right]$

(ii) Mean and variance.

**2.** A random variable X has the following probability function

| $X=x_i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(X=x_i)$ | 0 | K | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2 + k$ |

(i) Find the value of K (ii) $P(X < 6)$ (iii)$P(X \geq 6)$ (iv) $P(1 \leq x \leq 5)$

## SECOND HALF (All are 8- marks)

### (B) Continuous DISTRIBUTION:-

#### I- Uniform distribution

**1.** The moment generating function of a uniform distribution and find mean and variance

**2.** Buses arrive at a specified bus stop at 15-min intervals starting at 7 a.m that is 7a.m ,7.15am ,7.30am,…..etc. If a passenger arrives at the bus stop at a random time which is uniformly distributed between 7am and 7.30 am.Find the probability that he waits (i) less than 5 min (ii) atleast 12 min for bus.

**3.** Subway trains on a certain run every half an hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least 20-minutes.

#### II- Exponential distribution

***1.*** *The moment generating function of a exponential distribution and find mean and variance.* (i.e) $P(x) = \begin{cases} \lambda e^{-\lambda x}, x \geq 0, \lambda > 0 \\ 0 \quad , otherwise \end{cases}$

**2.** *State and prove the memory less property of the exponential distribution.*

**3.** The time in hours required to repair a machine is exponentially distribution with perimeter $\lambda = \frac{1}{2}$. (i) what is probability that the repair time exceeds 2 hours. (ii)What is the conditional probability that repair takes at least 10 hours given that is duration exceeds 9 hours.

**4.** The length of time a person speaks over phone follows exponential distribution with mean 6. What is the probability that the person will take for
(i) more than 8-minutes (ii) between 4 and 8 minutes.

## III- Normal distribution

1. The moment generating functions of a normal distribution and find mean and variance.
2. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and variance.
3. In a distribution exactly normal 75 of the items are under 35 and 39% are under 63. What are the mean and S.D of the distribution.
4. The peak temperature T, as measured in degrees Fahrenheit on a particularday is the Gaussian (85,10) random variables. What is $P(T > 100)$ , $P(T < 60)$, $P(70 < T < 100)$.
5. An electrical firm manufactures light bulbs that have a life before burn out that is normally distributed with mean equal to 800 hrs and a standard deviation of 40 hrs. Find the (i) the probability that a bulb more than 834 hrs. (ii) the probability that bulbs burns between 778 and 834 hrs.



### (C) Problems on continuous random variables:-

**1.** A continuous random variable X that can assume any value between X=2 and X=5 has a probability density function given by $f(x) = k(1 + x)$. Find $P(X < 4)$.

**2.** A continuous random variable X has pdf $f(x) = \begin{cases} \frac{k}{1+x^2}, -\infty < x < \infty \\ 0 \quad , otherwise \end{cases}$

Find (i) the value of K. (ii) $P(X > 0)$ (iii) distribution function of X.

**3.** If X is a continuous R.V's whose pdf is given by $f(x) = \begin{cases} c[4x - 2x^2], 0 < x < 2 \\ 0 \quad , otherwise \end{cases}$

**4.** The probability distribution function of a R.V's is $f(x) = \begin{cases} x\,, \ 0 < x < 1 \\ 2 - x \ , 1 < x < 2 \\ 0 \quad , \ x > 2 \end{cases}$.

Find the cumulative distribution function.

**5.** The density function of a R.V's X is given by $(x) = Kx(2 - x), 0 \leq x \leq 2$ .
Find the mean and variance.

**6.** The c.d.f of a R.V's X is $F(x) = 1 - (1 + x), x \geq 0$. Find the p.d.f of X, mean and variance.

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# UNIT-2 TWO-DIMENSIONAL RANDOM VARIABLES

## PART – A

1. If two random variables X and Y have probability density function (PPF) $f(x,y) = ke^{-(2x+y)}$ for x ,y >0,evaluate k.
2. Define joint probability distribution of two random variables X and Y and state its properties.
3. If the point pdf of (X,Y) is given by $f(x,y) = e^{-(x+y)}\, x \geq 0, y \geq 0$ find E[XY].
4. If X and Y have joint pdf $f(x,y) = \begin{cases} x+y, 0 < x < 1, 0 < y < 1 \\ 0, otherwise \end{cases}$, check whether X and Y are ndependent.
5. Find the marginal density functions of X and Y if $f(x,y) = \frac{2}{5}(2x+5y), 0 \leq x \leq 1, 0 \leq y \leq 1$.
6. If the function $f(x,y) = c(1-x)(1-y), 0 < x < 1, 0 < y < 1, y < 1$ to be a density function,find the value of c.
7. Let X and Y be continuous RVs with J.p.d.f $f(x,y) = \begin{cases} 2xy + \frac{3}{2}y^2, 0 < x < 1, 0 < y < 1 \\ 0, otherwise \end{cases}$ .Find P(X +Y<1).
8. The regression lines between twqo random variables X and Y is given by 3X +Y =10 and 3X +4Y =12. Find the co-efficient of correlation between X and Y.
9. If X and Y are random variables such that Y = aX +b where a and b are real constants, show that the correlation co-efficient r(X,Y) between that has magnitude one.
10. If Y = -2X +3 , find the cov(X,Y).
11. Let (X,Y) be a two dimensional random variable. Define covariance of (X,Y). If X and Y are independent , what will be the covariance of (X,Y).
12. The regression equations of X on Y and Y on X are respectively 5x –y =22 and 64x -45y =24. Find the means of X and Y.
13. The tamgent of the angle between the lines of regression Y on X and X on Y is 0.6 and $\sigma_x = \frac{1}{2}\sigma_y$ . Find the correlation coefficient.
14. State the central limit theorem for independenc and identically distributed random variables.
15. The two regression equations of two random variables X and Y are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. find mean values of X and Y.

16. The angle between the two lines of regression.

17. If X and y are independent random variables with variance 2 and 3, then find the variance of $3x + 4y$.

18. The lines of regression in a bivariate distribution are $x + 9y = 7$ and $y + 4x = \frac{49}{3}$. find the coefficient of correlation.

19. If the joint P.d.f of $(x, y)$ is $f(x, y) = \begin{cases} \frac{1}{4} ,0 \le x, y \le 2 \\ 0 \ \ ,\text{otherwise} \end{cases}$ Find $P[x + y \le 1]$

20. If two random variable X and Y have P.d.f $f(x, y) = Ke^{-(2x+y)}$, for $x, y \ge 0$. Find the value of K.

21. Find K if the joint p.d.f of a bivariate random variable is given by $f(x, y) = \begin{cases} k(1 - x)(1 - y), 0 < (x, y) < 1 \\ 0 \qquad ,\text{otherwise} \end{cases}$

22. 8. If the joint p.d.f of $(x, y)$ is $f(x, y) = \begin{cases} e^{-(x+y)}, & x>0,y>0 \\ 0 & ,\text{otherwise} \end{cases}$ Check whether X and Y are independent.

23. The joint p.m.f of a two dimensional random variable (x,y) is given by $\mathbf{P(x, y) = K(2x + y)}$, $\mathbf{x = 1, 2}$ and $\mathbf{y = 1, 2}$, where K is a constant. Find the value of K.

# PART – B

## FIRST HALF (All are 8- marks)

### *I- Problems on discrete random variable*

1. The joint PMF of two random variables X and Y is given by

   $P(x, y) = \begin{cases} k(2x + y), x = 1,2; y = 1,2 \\ 0 \quad ,\text{otherwise} \end{cases}$, where K is a constant.

   (i) Find the K. (ii) Find the marginal PMF of X and Y.

2. The joint probability mass function of (x,y) is given by $P(x, y) = \frac{1}{72}(2x + 3y)$ $x = 0,1,2$ $and$ $y = 1,2,3$. Find all the marginal and conditional probability distribution of X and Y.

3. The joint probability mass function of (x,y) is given by $P(x, y) = K(2x + 3y)$ $x = 0,1,2$ ; $y = 1,2,3$. (i) Find all the marginal and conditional probability distribution. Also find the probability distribution of $(x + y)$ $and$ $P[x + y > 3]$

4. The joint distribution of X and Y is given by $f(x, y) = \frac{x+y}{21}$ , $x = 1,2,3; y = 1,2$. Find the marginal distribution. Also find $E[xy]$.

5. Three balls are drawn at random without replacement from box containing 2- white , 3-red, 4-black balls. If X denote the no.of white balls and Y denote the no.of red balls drawn.Find the joint probability distribution of (X,Y)

## II- ***Problems on continuous random variable***

1. The joint PDF of (x,y) is $f(x,y) = e^{-(x+y)}, x, y \geq 0$. Are X and Y independent?
2. If the joint probability distribution function of a two dimensional random variable (x,y) is given by $f(x,y) = \begin{cases} (1-e^{x})(1-e^{y}), & x > 0, y > 0 \\ 0 & ,\text{otherwise} \end{cases}$. (i) Find the marginal densities of X and Y. Are X and Y independent? (ii) $P[1 < x < 3, 1 < y < 2]$.
3. Given the joint density function of X and y as $f(x,y) = \begin{cases} \frac{1}{2} x e^{-y}; 0 < x < 2, y > 0 \\ 0 \quad ,elsewhere \end{cases}$. Find the distribution X+Y.
4. The joint PDF of the random variables (x,y) is given by $f(x,y) = k\,xy\,e^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of K and also Prove that x and Y are independent.
5. If X and Y are two random variable having joint density function $f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), 0 < x < 2, 2 < y < 4 \\ 0 \quad , otherwise \end{cases}$ .Find (i) $P[x < 1 \cap y < 3]$ (ii) $P[x + y < 3]$ (iii) $P[x < 1 / y < 3]$.
6. Suppose the point probability density function is given by $f(x,y) = \begin{cases} \frac{6}{5}(x+y^2); 0 < x < 1, 0 < y < 1 \\ 0 \quad ,elsewhere \end{cases}$. Obtain the marginal P.d.f of X and Y. Hence find $P\left[\frac{1}{4} \leq y \leq \frac{1}{2}\right]$
7. Given $f_{xy}(x,y) = c\,x(x-y), 0 < x < 2; -x < y < x$,
   (i) Evaluate C (ii) Find $f_x(x)$ and $f_y(y)$ (iii) $f_{\frac{x}{y}}\left(\frac{x}{y}\right)$.

## III- ***Problems on correlation and covariance***

1. The joint PDF of a random variable (x,y) is $f(x,y) = 25e^{-5y}, 0 < x < 0.2, y > 0$. Find the covariance of x and Y.
2. Two random variables X and Y have the following joint p.d.f

   $f(x,y) = \begin{cases} 2 - x - y; 0 < x < 1, 0 < y < 1 \\ 0 \quad ,elsewhere \end{cases}$. (i) Find the $var(x)$ and $var(y)$

   (ii) The covariance between x and y. Also find $\rho_{xy}$.
3. If X and Y be discrete R.V's with p.d.f $f(x,y) = \frac{x+y}{21}, x = 1,2,3; y = 1,2$.
   (i) Find the mean and variance of X and Y, (ii) $cov(x,y)$ (iii) $r(x,y)$.

**4.** Two random variables X and Y have the following joint p.d.f $f(x,y) = \begin{cases} x+y\ ;\ 0<x<1\ ,0<y<1 \\ 0 \qquad ,elsewhere \end{cases}$. (i) Obtain the correlation co-efficient between X and y. (ii) Check whether X and Y are independent.

**5.** Find the coefficient of correlation between X and Y from the data given below.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| X : | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y : | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

**6.** Find the coefficient of correlation between industrial production and export using the following data.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Production (x): | 55 | 56 | 58 | 59 | 60 | 60 | 62 |
| Export (y) : | 35 | 38 | 37 | 39 | 44 | 43 | 44 |

## SECOND HALF (All are 8- marks)

### *I- Problems on regression line*

**1.** Two random variables X and Y are related as $y = 4x + 9$. find the correlation coefficient between X and Y.

**2.** The two lines of regression are $8x - 10y + 66$ ; $40x - 18y - 214 = 0$. The variance of X is 9. Find the mean values of X and Y. Also find the coefficient of correlation between the variables X and Y and find the variance of Y.

**3.** The two lines of regression are $8x - 10y + 66$ *and* $40x - 18y - 214 = 0$. Find the mean values of X and Y. Also find the coefficient of correlation between the variables X and Y.

**4.** The regression equations are $3x + 2y = 26$ *and* $6x + y = 31$. Find the correlation coefficient between x and y.

**5.** The equation of two regression lines are $3x + 12y = 19$ *and* $3x + 9y = 46$. Find $\bar{x}$ , $\bar{y}$ and the correlation coefficient between x and y.

## ***II- Transformation of random variables***

1. If X and Y are independent random variables with p.d.f $e^{-x}, x \geq 0$ ; $e^{-y}, y \geq 0$ respectively. Find the density function of $U = \frac{X}{X+Y}$ $and\ V = X + Y$.

   Are U &V independent.

2. The joint P.d.f of X and Y is given by $f(x,y) = e^{-(x+y)}, x \geq 0, y \geq 0$.

   Find the probability density function of $U = \frac{X+Y}{2}$

3. If X and Y are independent exponential distributions with parameter 1 then find the P.d.f of $U = X - Y$.

4. Let $(X, y)$ be a two-dimensional non-negative continuous random variable having the joint density $f(x,y) = \begin{cases} 4xy\, e^{-(x^2+y^2)}, & x \geq 0, y \geq 0 \\ 0 & , elsewhere \end{cases}$.

   find the density of $U = \sqrt{x^2 + y^2}$

5. If the p.d.f of a two dimensional R.V (x,y) is given by $f(x,y) = x + y$ ,$0 \leq (x,y) \leq 1$ . Find the p.d.f of $U = XY$.

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## UNIT-3 ANALYTIC FUNCTIONS

### PART-A

1. If $f(z) = z^2$ analytic? Justify
2. Show that $f(z) = |z|^2$ is differentiable at $z = 0$ but not analytic at $z = 0$.
3. Check whether $w = \bar{z}$ is analytic everywhere or not.
4. For what values of a,b,c the function $f(z) = (x - 2ay) + i(bx - cy)$ is analytic.
5. Find the constants a,b if $f(z) = x + 2ay + i(3x + by)$ is analytic.
6. Find the constants a,b,c if $f(z) = x + ay + i(bx + cy)$ is analytic.
7. Prove that $w = \sin 2z$ is analytic function.
8. Find the fixed point (or) Invariant point (or) critical point

   **i)** $w = \frac{1+z}{1-z}$ **ii)** $w = \frac{z-1}{z+1}$ **iii)** $w = \frac{6z-9}{z}$ **iv)** $w = 1 + \frac{2}{z}$ v) $f(z) = z^2$
9. Prove that the bilinear transformation has atmost two fixed point.
10. Show that $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is a harmonic.
11. Prove that $u = e^x \sin y$ is harmonic.
12. Find the value of 'm' if $u = 2x^2 - my^2 + 3x$ is harmonic.
13. Find the image of the circle $|z| = 3$ under the transformation $w = 2z$.

## PART-B

## I- Bilinear Transformation

1. Find the bilinear transformation of the points $\boldsymbol{Z = 1, i, -1}$ $into$ $\boldsymbol{W = 0, 1, \infty}$

2. Find the bilinear transformation of the points $\boldsymbol{Z = -1, -i, 1}$ $into$ $\boldsymbol{W = \infty, i, 0}$

3. Find the bilinear transformation of the points $\boldsymbol{Z = 0, 1, \infty}$ $into$ $\boldsymbol{W = i, 1, -i}$

4. Find the bilinear transformation of the points $\boldsymbol{Z = 1, i, -1}$ $into$ $\boldsymbol{W = i, 0, -i}$

5. Find the bilinear transformation of the points $\boldsymbol{Z = -1, 0, 1}$ $into$ $\boldsymbol{W = -1, -i, 1}$

6. Find the bilinear transformation of the points $\boldsymbol{Z = 1, i, -1}$ $into$ $\boldsymbol{W = 2, i, -2}$

7. Find the bilinear transformation of the points $Z = -i, 0, i$ $into$ $W = -1, i, 1$

8. Find the bilinear transformation of the points $Z = 0, -1, i$ $into$ $W = i, 0, \infty$

## II- Conformal Mapping

1. Find the image of $|\boldsymbol{Z - 1}| = \boldsymbol{1}$ under the map $\boldsymbol{w = \frac{1}{z}}$.

2. Find the image of $|\boldsymbol{Z - 1}| = \boldsymbol{1}$ under the map $\boldsymbol{w = \frac{1}{z}}$.

3. Find the image of $|\boldsymbol{Z + i}| = \boldsymbol{1}$ under the map $\boldsymbol{w = \frac{1}{z}}$.

4. Find the image of $|\boldsymbol{Z - 2i}| = \boldsymbol{2}$ under the map $\boldsymbol{w = \frac{1}{z}}$.

5. Find the image of $\boldsymbol{1} < y < 2$ under the tansformation $\boldsymbol{w = \frac{1}{z}}$

6. Find the image of the following region under the transformation $\boldsymbol{w = \frac{1}{z}}$

   i) the half plane $\boldsymbol{x} > c$ $when$ $\boldsymbol{c} > 0$ ii) the half $y > c$ $when$ $c < 0$

   iii) the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ iv) the infinite strip $\boldsymbol{0} < y < \frac{1}{2}$

7. Show that the transformation $w = \frac{1}{z}$ transforms all circles and straight lines in the **W- plane** into circles or straight lines in the **Z-plane.**

## III- Find the analytic functions $f(z) = u + iv$

1. Find an analytic function, whose real part is given $u(x, y) = \frac{\sin 2x}{\cosh 2y - \cos 2x}$
2. Find an analytic function, whose real part is given $u(x, y) = \frac{\sin 2x}{\cosh 2y + \cos 2x}$
3. Find an analytic function, whose real part is given $u(x, y) = e^x(x \cos y - y \sin y)$
4. Find an analytic function $f(z) = u + iv$ , if $u = e^{2x}(x \cos 2y - y \sin 2y)$
5. Find an analytic function $f(z) = u + iv$ , **if** $u = e^{-2xy} sin\left(x^2 - y^2\right)$
6. Find an analytic function $f(z) = u + iv\ if\ \ v = e^{-x}(x \cos y + y \sin y)$
7. If $f(z) = u + iv$ is analytic, find $f(z)$ given that $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$
8. Determine the analytic function $f(z) = u + iv$ if $u - v = e^x(\cos y - \sin y)$

## IV-Harmonic Functions

1. To prove that $u(x, y) = \frac{1}{2}\log(x^2 + y^2)$ is harmonic function and also find its conjugate harmonic.
2. Find the function $u(x, y) = x^2 - y^2$ and $v\,(x, y) = \frac{-y}{x^2+y^2}$ is harmonic function.
3. Prove that $e^x(x \cos y + y \sin y)$ can be the real part of an analytic function and determine its harmonic conjugate.

## V- Properties and Results

1. Prove that the real and imaginary parts of an analytic functions are harmonic.
2. If $f(z) = u(x,y) + iv(x,y)$ is an analytic function, show that the curves $u(x,y) = c_1 and\ v(x,y) = c_2$ Cut orthogonally.
3. An analytic $\boldsymbol{f(z) = u + iv}$ function with constants modulus is constants.
4. If $\boldsymbol{f(z)}$ is a regular function of Z , prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\boldsymbol{f(z)}|^2 = 4|\boldsymbol{f'(z)}|^2$.

**QUESTION BANK**

**PERIOD:** AUG – DEC-2022 **BATCH**: 2021-2025

**BRANCH:** EEE **YEAR/SEM:** II/03

**SUB CODE/NAME:** MA3303 –PROBABILITY AND COMPLEX FUNCTION

## UNIT 4 – COMPLEX INTEGRATION

### PART – A

1. State the Cauchy's integral theorem.
2. State the Cauchy's integral formulae.
3. State the Cauchy's residue theorem.
4. Define Singular point.
5. Define Isolated singular points.
6. Define Essential singular point.
7. Define Removable singular point.
8. Find the residue of $f(z) = \frac{z^2}{(z-2)(z+1)^2}$ $at\ z = 2$
9. Find the residue of $f(z) = \frac{z^2}{(z+2)(z-1)^2}$ $at\ the\ singular\ point\ z = 1$
10. Evaluate $\int_c \frac{z}{(z-2)} dz$ , $where\ C\ is\ the\ circle\ |z| = 1$
11. Evaluate $\int_c \frac{e^z}{z+1} dz$ , $where\ C\ is\ the\ circle\ \left|z + \frac{1}{2}\right| = 1$
12. Evaluate $\int_c \frac{e^{2z}}{z^2+1} dz$ , $where\ C\ is\ the\ circle\ |z| = \frac{1}{2}$
13. Find the residue of $\frac{1-e^{2z}}{z^4}$ $at\ z = 0$
14. Find the residue of $\frac{1-e^{-z}}{z^3}$ $at\ z = 0$
15. Find the residue of the function $f(z) = \frac{4}{z^3(z-2)}$ $at\ the\ simple\ pole.$
16. Find the residue of $f(z) = ze^{2/z}$ $at\ z = 0$
17. Find the residue of $f(z) = e^{1/z}$ $at\ z = 0$
18. Find the residue of $f(z) = \frac{\sin z}{z^4}$ $at\ z = 0$
19. Find the residue of $f(z) = z\cos\frac{1}{z}$ $at\ z = 0$
20. Find the poles of the function $f(z) = \frac{1}{(z+1)(z-2)^2}$ and find the residue at the simple pole.

## PART-B

### I- Find the Laurent's series.

1. Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the series in the regions
   $i)\ |z| < 2$ $\quad ii)\ |z| > 3$ $\quad iii)\ 2 < |z| < 3$ using Laurent's series

2. Expand $f(z) = \frac{7z-2}{z(z-2)(z+1)}$ in Laurent's series if the
   $i)\ |z| < 2$ $\quad ii)\ |z| > 3$ $\quad iii)\ 2 < |z| < 3$ $\quad iv)\ 1 < |z+1| < 3$

3. Expand in Laurent's series of $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$

4. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ $i)\ |z| < 1\ ii)|z| > 2\ iii)1 < |z| < 2\ iv)\ 0 < |z-1| < 1$

5. Expand $f(z) = \frac{z}{(z-1)(z-2)}$ in the region $\quad i)\ |z| < 1 \quad ii)|z| > 2$
   $iii)1 < |z| < 2 \quad iv)\ |z-1| < 1$

6. Expand $f(z) = \frac{z}{(z+1)(z+2)}$ in the region $\quad i)\ |z| < 1 \quad ii)|z| > 2$
   $iii)1 < |z| < 2 \quad iv)\ |z+1| < 1$

7. $f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region
   $i)\ |z| < 2\ ii)|z| > 3\ iii)2 < |z| < 3\ iv)\ |z+1| < 2$

### II- Cauchy's Integral Formulae

1. Evaluate $\int_c \frac{z^2+1}{z^2-1}dz$ where C is a circle of unit radius and centre at $Z = 1$.
2. Evaluate $\int_c \frac{z^2+1}{z^2-1}dz$ where C is a circle of unit radius and centre at $Z = -1$.
3. Using Cauchy's integral formula, evaluate
   $\int_c \frac{z+4}{z^2+2z+5}dz\ , where\ C\ \ is\ the\ circle\ |z+1-i| = 2$

4. Using Cauchy's integral formula, evaluate

$\int_c \frac{z+4}{z^2+2z+5} dz$ , *where* $C$ *is the circle* $|z+1-i| = 2$

5. Using Cauchy's integral formula, evaluate

$\int_c \frac{z}{(z-1)^2(z+2)} dz$ , *where* $C$ *is the circle* $|z| = \frac{3}{2}$

6. Using Cauchy's integral formula, evaluate

$\int_c \frac{z+1}{(z-3)(z-1)} dz$ , *where* $C$ *is the circle* $|z| = 2$

7. Using Cauchy's integral formula, evaluate

$\int_c \frac{z}{(z-2)^2(z-1)} dz$ , *where* $C$ *is the circle* $|z-2| = \frac{1}{2}$

8. Using Cauchy's integral formula, evaluate

$\int_c \frac{z+1}{z^2+2z+4} dz$ , *where* $C$ *is the circle* $|z+1+i| = 2$ **(or)** $|z+1-i| = 2$

9. Using Cauchy's integral formula, evaluate

$\int_c \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ , *where* $C$ *is the circle* $|z| = 3$

10. If $\int_c \frac{3z^2+7z+1}{z-a} dz$ , *where* $c$ *is* $|z| = 2$, find $f(3)$, $f(1-i)$, $f'(1-i)$

## III-Cauchy's Residue Theorem

1. Evaluate $\int_c \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ , *where* $C$ *is the circle* $|z| = 3$ using Cauchy's residue theorem.
2. Evaluate $\int_c \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)^2(z-2)} dz$ , *where* $C$ *is the circle* $|z| = 3$ using Cauchy's residue theorem.
3. Evaluate $\int_c \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)^2} dz$ , *where* $C$ *is the circle* $|z| = 3$ using Cauchy's residue theorem.
4. Evaluate $\int_c \frac{(z-1)}{(z-1)^2(z-2)} dz$ , *where* $C$ *is the circle* $|z-i| = 2$ using Cauchy's residue theorem.

5. Evaluate $\int_c \frac{(z-1)}{(z+1)^2(z-2)}dz$ , *where C is the circle* $|z-i| = 2$ using Cauchy's residue theorem.

6. Evaluate $\int_c \frac{z}{(z^2+1)^2}dz$ , *where C is the circle* $|z-i| = 1$ using Cauchy's residue theorem.

## IV-Contour Integration

### TYPE-I

1. Evaluate $\int_0^{2\pi} \frac{\cos 2\theta \;\; d\theta}{5+4\cos\theta}$ using contour integration.
2. Evaluate $\int_0^{2\pi} \frac{\cos 3\theta \;\; d\theta}{5-4\cos\theta}$ using contour integration.
3. Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\cos\theta}$ using contour integration.
4. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$ using contour integration

### TYPE-II

1. Evaluate by contour integration $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}$ .
2. Evaluate by contour integration $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2}$
3. Prove that $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{a+b}$ , $a > b > 0$ using contour integration.
4. Using contour integration ,evaluate $\int_0^{\infty} \frac{x^2dx}{(x^2+1)^2}$ $if\ a > 0$.
5. Using contour integration ,evaluate $\int_0^{\infty} \frac{x^2dx}{(x^2+1)(x^2+4)}$.

# ST.ANNE'S

**COLLEGE OF ENGINEERING AND TECHNOLOGY**

(An ISO 9001:2015 Certified Institution)

Anguchettypalayam, Panruti – 607106.

**QUESTION BANK**

**PERIOD:** AUG – DEC-2022 **BATCH**: 2021-2025

**BRANCH:** EEE **YEAR/SEM:** II/03

**SUB CODE/NAME:** MA3303 –PROBABILITY AND COMPLEX FUNCTION

## UNIT 5 – ORDINARY DIFFERENTIAL EQUATIONS

### PART – A

1. Solve $(D^2 + 5D + 6)y = 0$.
2. Solve $(D^2 + 4D + 6)y = 0$.
3. Find the Particular integral of $(D^2 - 4)y = e^{-4x}$.
4. Find the Particular integral of $(D^3 + 4D)y = sin2x$.
5. Solve $(D^2 + 1)y = 0$.
6. Solve $(D^2 + 6D + 9)y = 0$.
7. Find the particular integral of $(D^2 + 4)y = cos2x$.
8. Find the particular integral of $(D^2 - 4)y = e^{-4x}$.
9. Reduce $(\boldsymbol{x^2D^2 - 3xD + 3)y = x}$ into a differential equation with constant coefficient.
10. Reduce $(\boldsymbol{x^2D^2 + xD + 1)y = 0}$ into a differential equation with constant coefficient.
11. Reduce $\boldsymbol{((2x+3)^2D^2 - (2x+3)D - 12))y = 6x}$ into a differential equation with constant coefficient.
12. Find the particular integral of $\boldsymbol{y'' - 6y' + 9y = 2e^{3x}}$.
13. Find the particular integral of $(D-1)^2 y = e^x \sin x$ **(Jan-18)**
14. Convert $x^2 y'' - 2xy' + 2y = 0$ into a linear differential equation with constant coefficients. **(Jan-18)**
15. Solve $(D^2+6D+9)\,y=0$
16. Find the particular integral of $\left(D^2-2D+1\right)y=\cosh x$
17. Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=0$

**18.** Transform the differential equation $\left(x^2D^2+4xD+2\right)y=x+\frac{1}{x}$ into a differential equation with constant coefficient.

**19.** Transform the equation $(2x-1)^2y''-4(2x-1)y'+8y=8x$ into the linear equation with constant coefficients.

**20.** Find the differential equation of x(t) given $\frac{dy}{dt}+x=\cos t, \frac{dx}{dt}+y=e^{-t}$

21. Solve $(D^3+1)y=0$. **(Nov18)**

22. Transform the equation $xy''+y'+1=0$ into the linear equation with constant coefficients **(Nov18)**

23. Find P.I of $(D-a)^2y=e^{ax}sinx$ **(Apr19)**

24. Solve $x^2y''xy'+y=0$ **(Apr19)**

**PART – B**

**Homogeneous equations**

1. Solve the differential equation $(D^2+3D+2)y=e^{-3x}$.
2. Solve the differential equation $(D^2-4D+4)y=e^{2x}+cos2x$.
3. Solve the differential equation $(D^2-3D+2)y=sin3x$.
4. Solve the differential equation $(D^2-5D+6)y=x^2+3$.
5. Solve the differential equation $(D^2+5D+4)y=e^{-x}sin2x$.
6. Solve the differential equation $(D^2+2D+1)y=e^{-x}x^2$.
7. Solve the differential equation $(D^2+4)y=xsinx$.
8. Solve the differential equation $(D^2+4D+5)y=e^x+x^2+cos2x+1$. **(Nov18)**

**Euler's and Legendre's equations**

9. Solve $(x^2D^2+4xD+2)y=logx$.
10. Solve $((x+1)^2D^2+(x+1)D+1)y=4\cos[\log(x+1)]$. **(Jan-18)**
11. Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin(\log(1+x))$
12. Solve $(x^2D^2-2xD-4)y=x^2+2\log x$
13. Solve $(x^2D^2-xD+1)y=\left[\frac{logx}{x}\right]^2$ **(Nov18)**
14. Solve $(2+x)^2\frac{d^2y}{dx^2}-(2+x)\frac{dy}{dx}+y=3x+4$ **(Apr19)**

## Variation of parameter

15. Solve $(D^2 + 1)y = cosecx$.

16. Solve $(D^2 + a^2)y = tanax$. **(Apr19)**

17. Solve $(D^2 + 4)y = sec2x$.

18. Solve the equation $(D^2 + 1)y = x\sin x$ by the method of variation of parameters.

19. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = x^2 \sin(\log x)$

20. Solve $y'' - 4y' + 4y = (x+1)e^{2x}$ **(Nov18)**

## Simultaneous equations

21. Solve the simultaneous equations Dx+y=sin2t and –x+Dy=cos2t. **(Jan-18)**

22. Solve $\frac{dx}{dt} + 4x + 3y = t; \frac{dy}{dt} + 2x + 5y = e^{2t}$

23. Solve $\frac{dx}{dt} + 5x - 2y = t$; $\frac{dy}{dt} + 2x + y = 0$

24. Solve the simultaneous equation $\frac{dx}{dt} - y = t$ and $\frac{dy}{dt} + x = t^2$ given $x(0) = y(0) = 2$.

25. Solve $\frac{dx}{dt} - \frac{dy}{dt} + 2y = cos2t$ , $\frac{dx}{dt} - 2x + \frac{dy}{dt} = sin2t$ **(Nov18)**

26. Solve $\frac{dx}{dt} + \frac{dy}{dt} + 3x = sint$ , $\frac{dx}{dt} + y - x = cost$ **(Apr19)**

27. Solve $(D^2 + 2D + 1)y = e^x sin2x$ by using the method of undetermined coefficients.

**(Apr19)**

28. Solve $(D^2 - 2D)y = 5\, e^x cosx$ by using the method of undetermined coefficients

**(Nov18)**